Multi-scale approaches for architectured and microstructured materials and metamaterials

Nantes, France, October 13-18, 2025

Summer School coordinated by

Angela Madeo
TU Dortmund, Germany

Jean-François Ganghoffer
Université de Lorraine, Nancy, France

Supported by
Université franco-allemande Deutsch-Französische Hochschule and European Commission H2020

Local scientific and organizational support

Giulio Sciarra
Ecole Centrale de Nantes, France

Book of Abstracts

Time	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	October, 13	October, 14	October, 15	October, 16	October, 17	October, 18
09:00 – 10.30	Angela Madeo	Samuel Forest	Bruno Lombard	Jean-François Ganghoffer	Varvara Kouznetsova	Ingo Münch
11:00 – 12:30	Patrice Cartraud	Patrice Cartraud	Patrizio Neff	Jean-François Ganghoffer	Giulio Sciarra	Ingo Münch
			Lunch break			
14:00 – **15:30**	Manuell Collet	Varvara Kouznetsova	Angela Madeo	Giuseppe Rosi	Marco Valerio d'Agostino	Marco Valerio d'Agostino
16:00 – **17:30**	Patrizio Neff	Manuell Collet	Bruno Lombard	Giuseppe Rosi	Samuel Forest	
18:00	Welcome aperitif		Conference dinner			

On the use of the asymptotic expansion method for the multiscalev modeling of heterogeneous elastic materials and structures: from lower-order to higher-order models.

Patrice Cartraud, Nantes Université, Ecole Centrale Nantes, France

The asymptotic expansion method is a powerful and rigorous approach for deriving simplified models of heterogeneous materials and structures [1-4]. This method uses as a starting point the 3D elastic problem and exploits a small parameter that characterizes the multiscale nature of the problem. For heterogeneous materials, this parameter represents the relative size of the heterogeneities compared to the size of the 3D domain. For beam-like structures, the parameter is the inverse of the slenderness. The asymptotic expansion method then leads to a series of microscopic problems, defined on a 3D domain, and to macroscopic problems, posed either on a 3D domain with homogeneous behavior or on 1D domain. The macroscopic behavior is determined from the solutions of the microscopic problems. In this presentation, various examples will be discussed to illustrate the use of the asymptotic expansion method. The focus will be on the meaning and limitations of the lower-order approximations, e.g. [6], as well as the construction of higher-order solutions and edge effects. An application in the field of nonlinear elasticity will also be presented [7].

[1] Kalamkarov, A. L., Andrianov, I. V., & Danishevs' kyy, V. V. (2009). Asymptotic homogenization of composite materials and structures.
[2] Kanouté, P., Boso, D. P., Chaboche, J. L., & Schrefler, B. (2009). Multiscale methods for composites: a review. Archives of Computational Methods in Engineering, 16, 31-75.
[3] Lewinski, T., & Telega, J. J. (2000). Plates, laminates and shells: asymptotic analysis and homogenization (Vol. 52). World Scientific.
[4] Sab, K., & Lebée, A. (2015). Homogenization of heterogeneous thin and thick plates. John Wiley & Sons.
[5] Trabucho, L., & Viano, J. M. (1996). Mathematical modelling of rods. Handbook of numerical analysis, 4, 487-974.
[6] Ameen, M. M., Peerlings, R. H., & Geers, M. G. (2018). A quantitative assessment of the scale separation limits of classical and higher-order asymptotic homogenization. European Journal of Mechanics-A/Solids, 71, 89-100.
[7] Ménard, F., & Cartraud, P. (2021). Solid and 3D beam finite element models for the nonlinear elastic analysis of helical strands within a computational homogenization framework. Computers & Structures, 257, 106675.

ADAPTIVE METACOMPOSITE FOR VIBRATION CONTROL

Manuel Collet, French National Centre for Scientific Research, Paris, France

Key Words: *Non-reciprocity, waves, Metacomposite, adaptive vibroacoustic control*

Research activities in smart materials and adaptive structures are of a great interest for many years. New technologies are now available for implementing smart cells inside architected material allowing us to design and construct a new class of smart materials that we can call "programmable metacomposites". These new generalized composites allow integration of dense and distributed set of smart materials, electronics, chip sets and power supply system inside material, for implementing distributed control strategies. This paves the way to imagine new strategies to control vibroacoustic flow in a large frequency band and implement unconventional behavior such as non-reciprocal wave propagation.

After a state of art of this domain of research, the wave diffusion control by means of piezoelectric metacomposites is firstly presented and analysed. Associated numerical method for modelling and optimizing are deeply analysed. Concepts of energy lensing and mechanical diode are also presented.

Plastic instabilities in periodic and random architectured materials

Samuel Forest, Paris Sciences et Lettres University, France

Porous metals offer remarkable architectural possibilities for optimizing the lightness and ductility of components. We first consider periodic lattices made of a shape-memory alloy, and study the propagation of Lüders-type plastic instabilities through these lattices. In experiments and simulations, both propagative and more homogeneous macroscopic modes are observed [1].

The Lüders phenomenon in these alloys is associated with the propagation of shear bands and the presence of a plateau on the constituent's tensile curve. What about lattice plasticity? We will see that the plateau can be preserved or eliminated by the chosen architecture among triangular, square lattices or honeycombs.

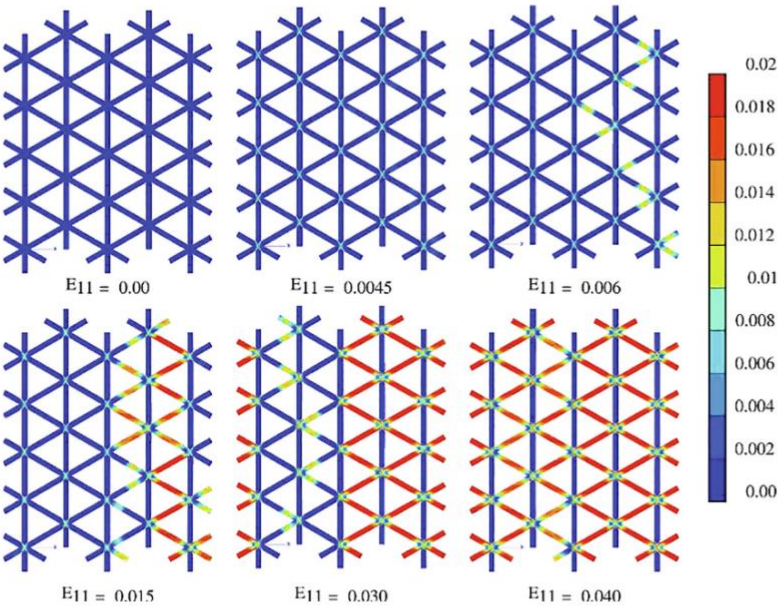
Metal foams appear more like a random arrangement of open or closed cells. Here, we study the ductility of nickel foams used in mobile phone and laptop batteries. The combination of X-ray micro-tomography and intensive finite element computation is used to describe crack initiation and propagation in detail. We use generalized continuum mechanics to propose an effective elastoplastic foam model for structural design [2]. The characteristic length of the proposed micromorphic medium is related to the cell size.

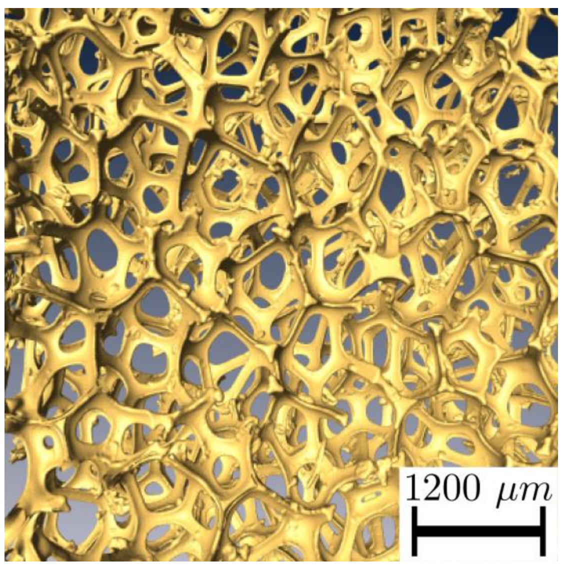
In both cases, anisotropy plays a significant role and must be accounted for in the continuum mechanical modelling.

References

[1] Antoine-Emmanuel Viard, Justin Dirrenberger and Samuel Forest, *Propagating material instabilities in planar architectured materials*, International Journal of Solids and Structures, vol. 202, pp. 532-551, 2020, 10.1016/j.ijsolstr.2020.05.027

[2] Thierry Dillard, Samuel Forest and Patrick Ienny, *Micromorphic continuum modelling of the deformation and fracture behaviour of nickel foams* , European Journal of Mechanics A/Solids, vol. 25, pp. 526-549, 2006. 10.1016/j.euromechsol.2005.11.006





Periodic homogenization towards generalized continua in static and dynamic conditions and applications to architectured materials

Jean-François Ganghoffer – Université de Lorraine, Nancy, France

The talk aims to provide a classification of generalized continua constructed by a micromechanical approach, relying on an extension of the celebrated Hill macrohomogeneity condition for the micromorphic effective medium. Starting from the Cauchy balance equations holding at the microscopic level, equilibrium equations for a micromorphic effective medium are derived. The construction of the static macroscopic variables conjugated to the introduced macroscopic kinematic degrees of freedom relies on the identification of the homogeneous contribution, representing in an exact manner the effective micromorphic medium, corrected by a fluctuation as a measure of its deviation from the initially heterogeneous periodic medium. The homogenized micromorphic constitutive law is derived based on suitable variational principles and the solution of the unit cell BVP's. Subclasses of generalized continua are derived by suitable projections of the micromorphic degree of freedom into reduced degrees of freedom. Examples of architected media representative of some of the constructed effective generalized media illustrate the proposed homogenization method. The ranking of the different effective media w.r.t. the small-scale parameter (ratio of lattice size to a macroscopic dimension) is established, based on the asymptotic expansion of the homogeneous part of the microscopic displacement. Based on a surface formulation of the extended Hill macrohomogeneity condition, edge effects in the response of lattice structures are quantified. The bending moment to curvature relation involves a classical term and a micropolar bending moment, the importance of which is decreasing asymptotically up to a nil value by increasing the number of unit cells in the height of the macrostructure. In the second part of the lecture, we extend the micromorphic framework to dynamics by deriving the macroscopic kinetic energy from the upscaling of microscopic classical principles. Our methodology focuses on identifying a homogeneous velocity that captures the microscale kinematics of the micromorphic effective medium, defined by the macroscopic velocity and the rates of the displacement gradient, distortion tensor, and its first gradient. Unit cell boundary value problems are formulated for the velocity fluctuation that leads to the computation of the effective dynamical micromorphic properties. The predictions of kinetic energy, incorporating higher-order contributions, are exemplified by the square and tetrachiral unit cells, emphasizing the crucial role of structural design in micromorphic media.

References

Alavi et al., 2021. Construction of micromorphic continua by homogenization based on variational principles. J. Mech. Phys. Solids, 153, 104278.
Reda, et al, 2021. Homogenization towards chiral Cosserat continua and applications to enhanced Timoshenko beam theories. Mechanics of Materials, 155, 103728.
Alavi et al., 2022. Continualization method of lattice materials and analysis of size effects revisited based on Cosserat models. Int. J. Solids Struct., 254–255 (2022) 111894.
Mawassi et al., 2022. Analysis of surface effects based on first and second strain gradient mechanics. Mech. Mat., 175(13): 104462.
Ganghoffer et al., 2025. Dynamical homogenization of microstructured media towards micromorphic effective continua. Int. J. Engng Sci., 211, 104255.

Modeling microstructure evolution and fluid transport in porous media via formal asymptotic expansions

Giulio Sciarra, École Centrale de Nantes, France

Porous solids are a very special kind of materials, strongly heterogeneous and capable to exhibit complex response to applied loading at the continuum scale, because of the evolution of their intrinsic microstructure. This is what is commonly known as material remodeling induced by pore transformation which altering the force chain network at the microscopic scale yields significant modifications of the response of the macroscopic continuum. The presence of fluids eventually flowing through the porous space, and having chemical or mechanical interactions with the pore walls, can also trigger local events which, modifying the current obstruction to the hydrodynamical flow, can induce modifications of the macroscopic characteristics of the porous continuum.
One can refer, for instance, to the processes of mineral solubility of the pore walls into the solution saturating the pore network or to the converse process of mineral precipitation, both induced by geochemical reactions. These are the typical phenomena possibly occurring in geothermal power plants when water is injected into reservoir rocks saturated in brine altering the chemical composition of the rock. A chemical disequilibrium between the underground fluid and its host rock arises, yielding one of the two above-mentioned processes. A similar phenomenon but driven by mechanical interactions between a granular skeleton and a fluid flowing through its porous network is that of internal erosion, where shear forces and pressure differences at the contact between the solid and the fluid can induce either fine particles detachment from or fine particles deposition on the solid grains. This phenomenon is critical for a large range of engineering applications. In petroleum industry sand production caused by erosion can block the transport of the hydrocarbon produced into the wellbore. According to civil infrastructures like earth dams, internal erosion, in particular backward erosion and suffusion, can progressively reduce the strength and modify the hydraulic conductivity of the core wall leading to the failure of the whole structure.
Referring to such kind of phenomena we aim at investigating the effects on the behavior of a saturated porous material of an evolving microstructure induced by the mass exchange between the solid and the fluid phases saturating the porous network, using two-scale asymptotic expansions. A thermodynamically consistent model of the fluid physics flowing through the porous network is proposed first, describing microstructure variations to be captured implicitly via the level set method. Adopting a similar approach as [1,2] the two-scale asymptotic expansions method is then applied to obtain an upscaled model capable to account for mass transfer. This last is proven to depend not only on the gradient of the macroscopic forces, such as the fluid pressure and the chemical potential, but also on the average velocity of the solid–fluid interface. More details are reported in [3].

References

[1] van Noorden, T.L.: Crystal precipitation and dissolution in a porous medium: effective equations and numerical experiments. Multiscale Model. Simul. 7(3), 1220–1236 (2009).
[2] Bringedal, C., Berre, I., Pop, I.S., Radu, F.A.: Upscaling of non-isothermal reactive porous media flow with changing porosity. Transp. Porous Media 114(2), 371–393 (2016).
[3] Rousseau, Q., & Sciarra, G. (2024). Microstructure evolution and fluid transport in porous media: a formal asymptotic expansions approach. Archive of Applied Mechanics, 94(11), 3253-3291.

Computational homogenization for locally resonant metamaterials

Varvara Kouznetsova – Eindhoven University of Technology, Netherlands

The past two decades have been marked by the ever increased interest into the development of various types of metamaterials. In particular, a class of metamaterials designed to manipulate the propagation of elastic/acoustic waves in the subwavelength regime, based on the local resonance principle, opens unprecedented possibilities for a broad class of applications. Analytical and numerical techniques for the analysis of wave dispersion in infinite metamaterial structures are well established (Bloch analysis, transmission spectra etc.). However, design of metamaterials for their integration in actual engineering systems requires modelling techniques that are applicable to finite size, complex shape domains, including interfaces and non-trivial, non-harmonic loads. In this talk, the transient computational homogenization methodology will be presented. The approach is applicable to general non-linear problems [1], while in linear cases it may be reduced to provide the closed-form homogenized effective constitutive relations [2]. After discussing the main framework, several applications and extensions will be addressed, e.g. application to the multi-layered metamaterial setups, incorporation of non-classical damping arising from visco-elastic behaviour of one or multiple metamaterial constituents, and dissipative fluid flow through porous structure, including local resonances in the fluid and/or solid [3,4].

[1] T.F.W. van Nuland, P.B. Silva, A. Sridhar, M.G.D. Geers and V.G. Kouznetsova, Transient analysis of nonlinear locally resonant metamaterials via computational homogenization. Mathematics and Mechanics of Solids, 24, pp. 3136–3155, 2019.

[2] A. Sridhar, V.G. Kouznetsova and M.G.D. Geers, Homogenization of locally resonant acoustic metamaterials towards an emergent enriched continuum. Computational Mechanics, 57, pp. 423–435, 2016.

[3] R. Liupekevicius, J.A.W. van Dommelen, M.G.D. Geers and V. G. Kouznetsova, An efficient multiscale method for subwavelength transient analysis of acoustic metamaterials. International Journal for Numerical Methods in Engineering, 125, e7505, 2024

[4] R. Liupekevicius, J.A.W. van Dommelen and M.G.D. Geers, V. G. Kouznetsova, Transient computational homogenization of heterogeneous poroelastic media with local resonances. Philosophical Transactions of Royal Society A, 382, 20230368, 2024

Wave propagation in non-reciprocal microstructured media: nonlinear contacts and space-time modulation

Bruno Lombard – French National Centre for Scientific Research, Marseille, France

Wave propagation is usually a reciprocal phenomenon: if you interchange source and receiver, the transmitted wave is the same. Breaking the reciprocity, for example, makes it possible to force wave propagation in one direction only, as in a diode. But the conditions required for this are highly restrictive: nonlinearities must be introduced, for example, or time invariance must be violated. In this course, we propose to study waves in microstructured media with these conditions, using asymptotic double-scale techniques. Initially, we'll consider a linear elastic medium with nonlinear contact conditions. By homogenization, we obtain an effective nonlinear medium, analogous to usual models of compressible fluid mechanics [1]. Next, we consider a medium with wave-like spatio-temporal modulation. Contrary to what is commonly thought, we will show by second-order homogenization that it is sufficient to modulate a single physical parameter to obtain non-reciprocity [2]. Finally, the case of time-modulated jump conditions will be studied [3]. Depending on the scaling between the wave center frequency and the modulation frequency, different regimes will be obtained by homogenization. Parametric instability or Willis-type coupling will be demonstrated.

[1] C. Bellis, B. Lombard, M. Touboul, R. Assier, "Effective dynamics for low-amplitude transient elastic waves in a 1D periodic array of non-linear interfaces", *Journal of the Mechanics and Physics of Solids* 149 (2021), 104321.
[2] M. Touboul, B. Lombard, R. Assier, S. Guenneau, R. Craster, "High-order homogenisation of the time-modulated wave equation: non-reciprocity for a single varying parameter", *Proceedings Royal Society London A* 480 (2024), 20230776.
[3] M. Darche, R. Assier, S. Guenneau, B. Lombard, M. Touboul, "Wave propagation across a time-modulated imperfect interface", submitted to Comptes-Rendus Mécaniques (2025).

Relaxed micromorphic modeling of boundary effects in finite-size metamaterials through the new concept of macroscopic interface forces

Angela Madeo – Technical University Dortmund, Germany

In these lectures, I will show how the relaxed micromorphic model can be used to describe the dynamical behavior of anisotropic mechanical metamaterials. I will show to which extent the proposed model is able to capture all the main macroscopic dynamical characteristics of the targeted metamaterials, namely, stiffness, anisotropy, dispersion and band-gaps.
The simple structure of our material model, which simultaneously lives on a micro-, a meso- and a macroscopic scale, requires only the identification of a limited number of frequency-independent parameters, thus allowing the introduction of pertinent boundary conditions to be imposed at macroscopic metamaterials' boundaries when the model is framed in the context of Variational Principles.
I will show how this modelling approach can be applied to the study of the scattering properties of finite-size metamaterials' through the introduction of the new concept of "macroscopic interface forces".
More particularly, I will introduce for the first time the concepts of non-coherent interfaces and microstructure-driven interface forces in the framework of micromorphic elasticity.
I will show that such concepts are of paramount importance when studying the response of finite-size mechanical metamaterials at the homogenized macro-scale. The need of introducing interface forces is elucidated through numerical examples comparing reduced relaxed micromorphic simulations to their full-microstructured counterparts. These results provide a milestone for the understanding of metamaterials' modeling at the homogenized scale and for the use of micromorphic-type models to achieve an accurate upscaling towards larger-scale metamaterials' structures.
These new results are part of the ERC consolidator Grant META-LEGO n° 101001759 and will be exposed during the forecasted lectures.

The mathematics of generalized continua in the static case.

Patrizio Neff – University of Duisburg-Essen, Germany

I consider linear models of generalized continua (micromorphic, relaxed micromorphic, cosserat etc) including classical linear elasticity. I develop the mathematical existence theory for the corresponding boundary value problems in the static case. These investigations are the basis for a correct treatment in the context of the finite element methods. Necessary mathematical requisites are introduced but these should foster understanding and not detailed calculus. Of primary importance are Korns and generalized Korns inequalities. They will be motivated and proved in simple cases.
I will give an outlook on the modelling and mathematical treatment of the finite strain case as well.

Elastic wave propagation in the framework of strain gradient elasticity

<u>Giuseppe Rosi</u> – Paris-Est Créteil University, France

The study of elastic wave propagation is a fundamental tool in different fields, from Non-destructive Damage Evaluation (NDE) to ultrasonic imaging. Usually, NDE and characterisation techniques rely on inversion methods based on homogenised theories, that are valid only when the wavelength of the perturbation is considerably larger than the characteristic size of the heterogeneities of the materials. When the wavelength approaches this characteristic size, an upscaling occurs, and microscopic effects can be transferred to the macro-scale. In this case, classic models used in the aforementioned inversion procedures can fail to predict the correct response [1] and they need to be improved [2,3].
In this talk, we address unit cells with high order rotational symmetries, such as honeycombs, or those that do not have any centre of inversion (non-centrosymmetric) nor symmetry plane (chiral). It will be shown that unconventional effects, such as onset of anisotropy or acoustical activity, can be observed. We will also demonstrate that accurately describing these materials as an equivalent homogeneous continuum requires the application of an enriched or generalized theory, such as strain gradient elasticity. Moreover, the analysis of the generalised acoustic (or Christoffel) tensor defined in this framework can give a useful insight on the dynamic features of the architectured material.

References

[1] Rosi, G., Auffray, N. and Combescure, C. “On the Failure of Classic Elasticity in Predicting Elastic Wave Propagation in Gyroid Lattices for Very Long Wavelengths”, Symmetry, 12, 1243 (2020).
[2] Rosi, G. and Auffray, N. “Continuum modelling of frequency dependent acoustic beam focussing and steering in hexagonal lattices”, European journal of Mechanics/A Solids, 77, (2019).
[3] Rosi, G., Auffray, N. and Combescure, C. “Elastic wave propagation in cubic non-centrosymmetric and chiral architectured materials: insights from strain gradient elasticity”, Eutopean Journal of Mechanics/A Solids (2024).